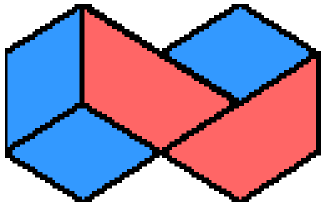
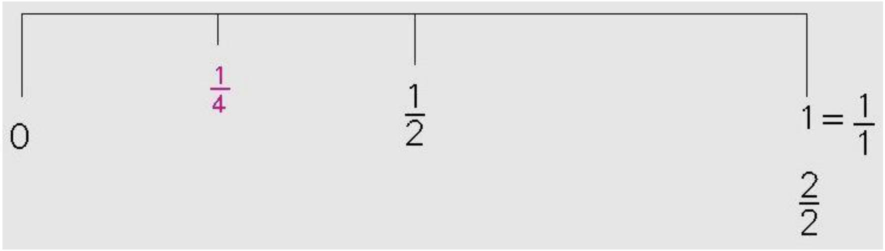
Teaching and Learning Fractions in the Middle Years

(TQI program: 003658)

27 October 2018

Which one is the Odd One Out and how do you know?

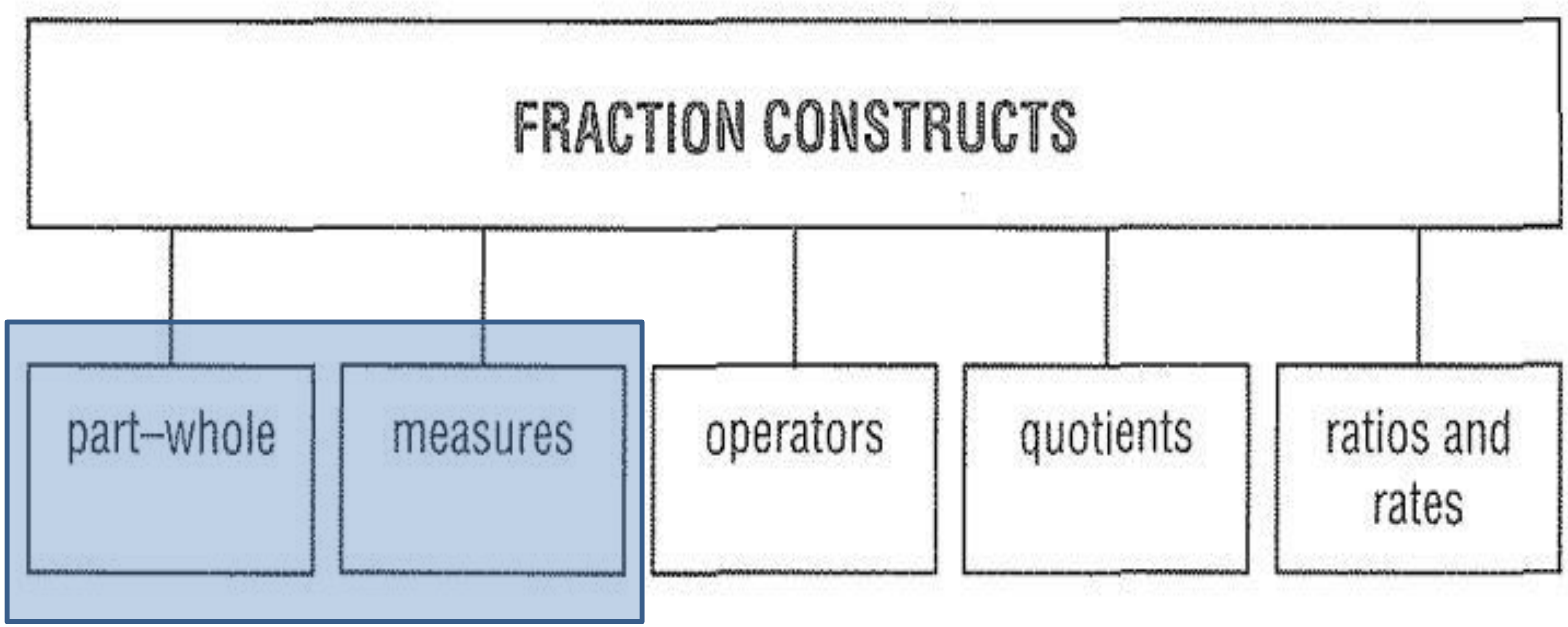




Elevating Learning

Leonie Anstey and Matt Sexton

Fraction constructs



Today's professional learning focus

(Clarke, 2006)

Fraction key ideas

Quantity

Number triad relationships

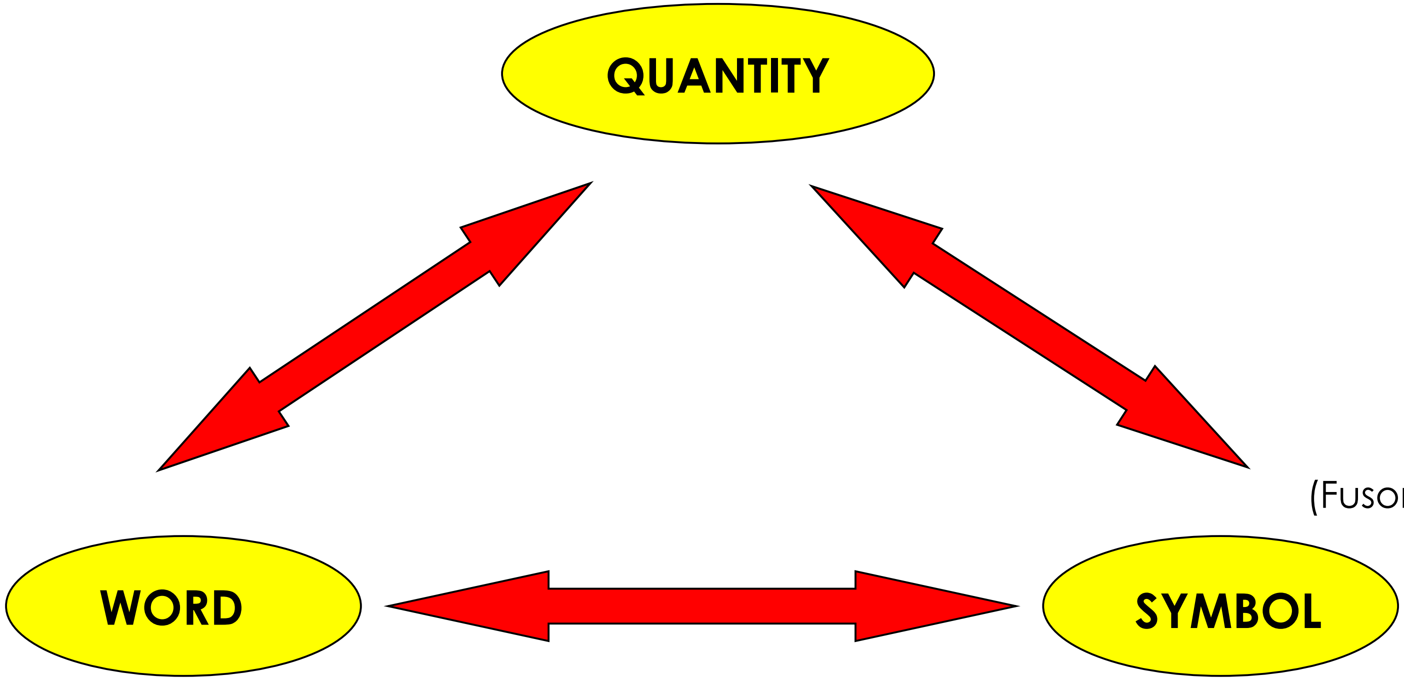
Partitioning

Equivalence

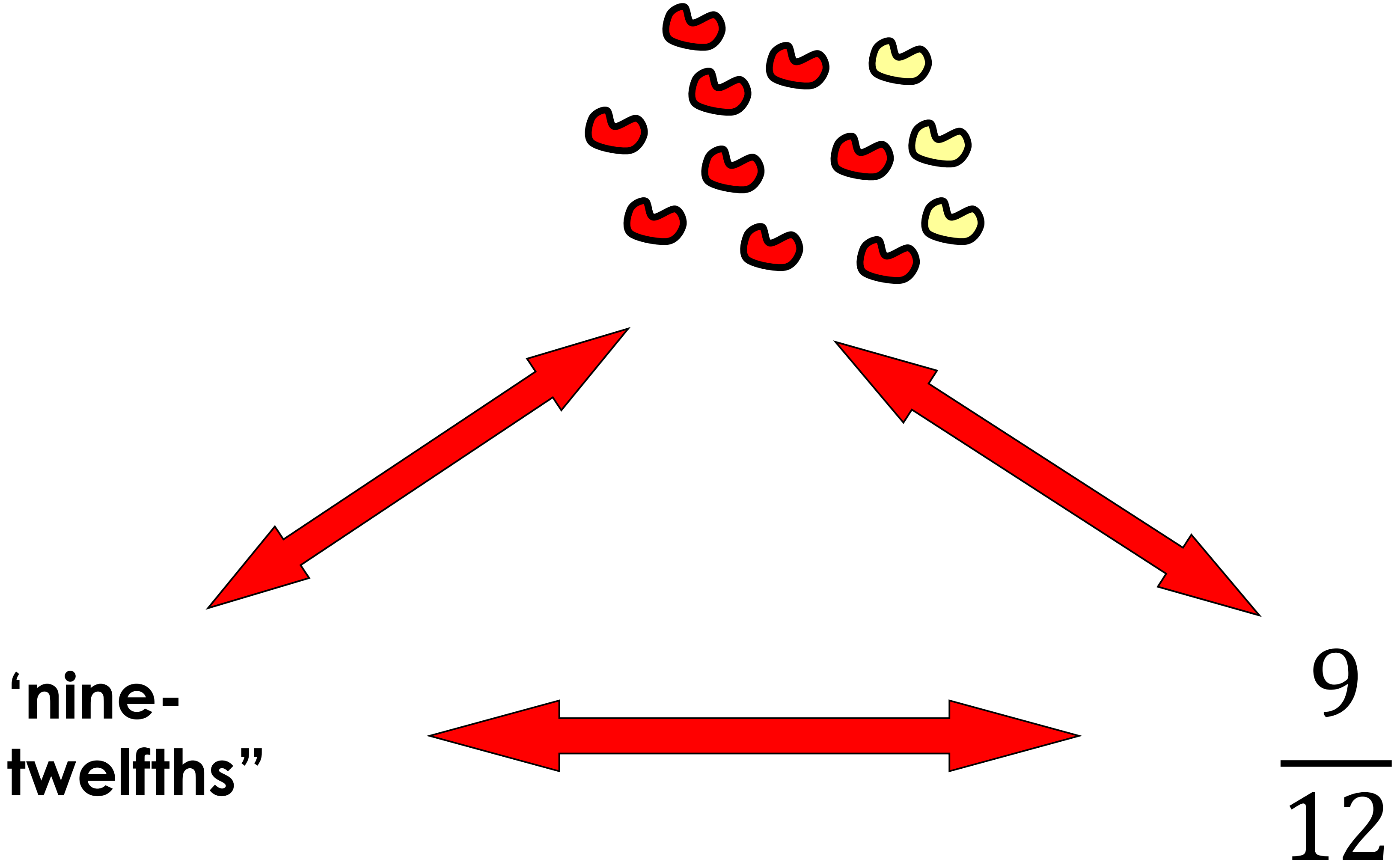
Benchmarking

Number triad

Students must connect the three pieces of information when understanding and interpreting numbers, especially rational numbers like fractions



(Fuson et al., 1997)

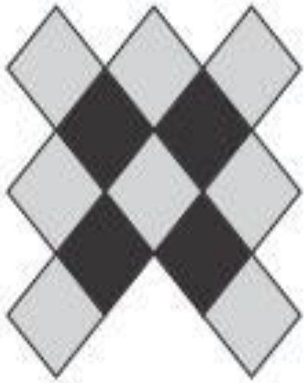


We also need to understand that the same quantity, can have different names.

“nine-twelfths is also known as “three-quarters” (or three-fourths)” and also ….

Year 5 NAPLAN (2008)
Fractions

37 This shape was made using 12 tiles.

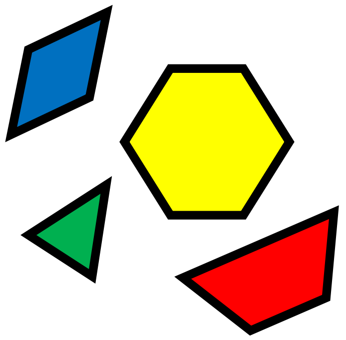


What fraction of the tiles in the shape is black?

$\frac{4}{8}$ ○ $\frac{1}{3}$ ○ $\frac{1}{4}$ ○ $\frac{1}{8}$ ○

Percentage correct National
23

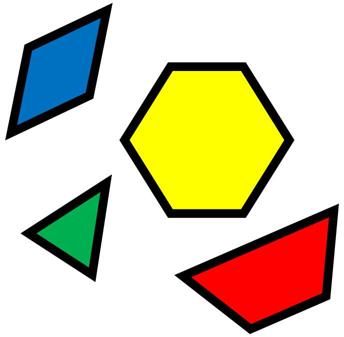
Pattern Block fractions



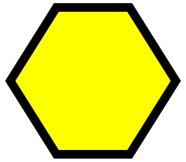
Key ideas:
- Fraction as part-whole (fraction of an area model)
- Equivalence
- Quantity
- Partitioning

What are the blocks and what are they called?

Pattern Block fractions

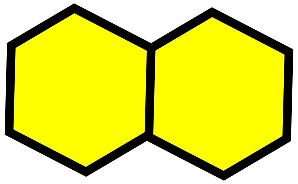


How many different congruent hexagons can you find?

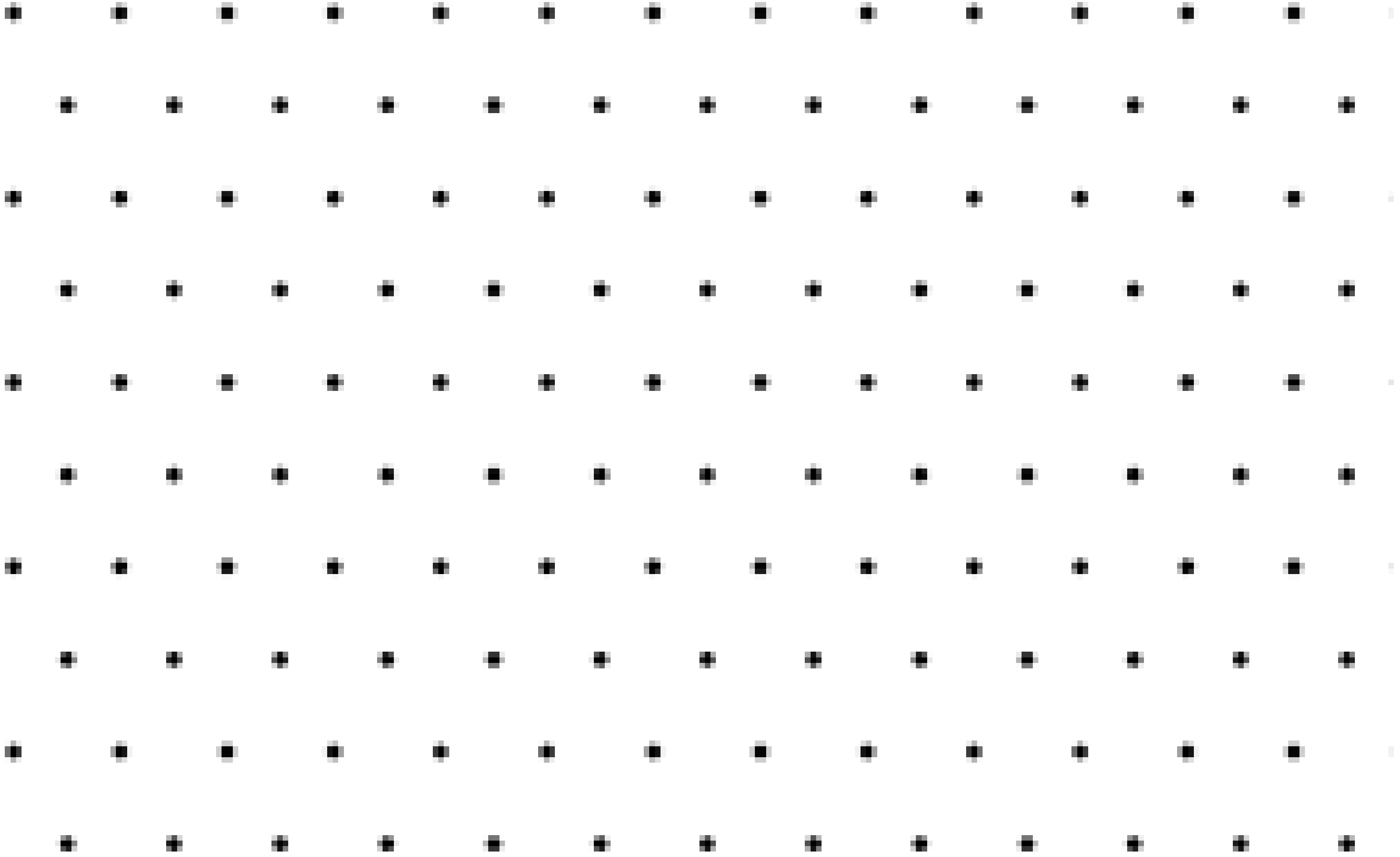


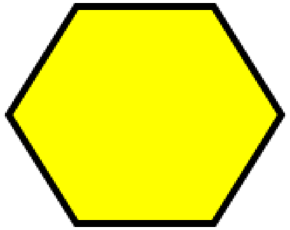
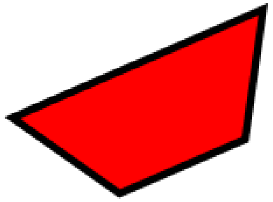
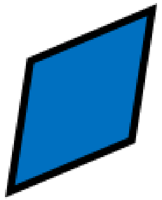
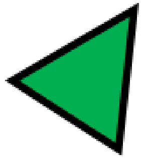
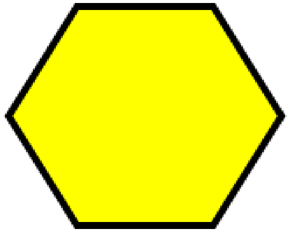
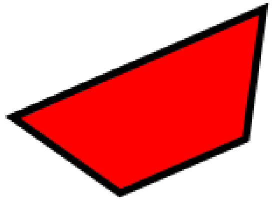
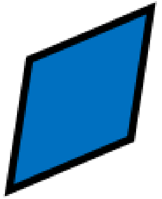
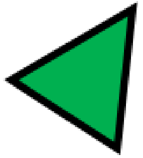
If the hexagon has a value of 1, how could you describe each of the parts?

If two hexagon has a value of 1, how could you describe each of the parts?

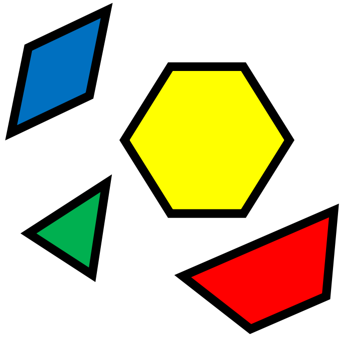


If ….has a value of 1, how could you describe each of the parts?



If these are 1 What fraction are these?				
				
				
				
				

Pattern block designs



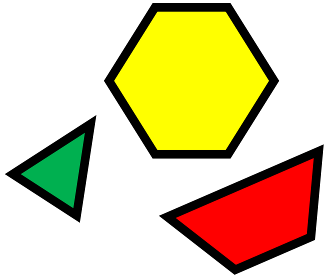
Create a design using any of the pattern blocks where one-third is represented

- Is there another way?

- How many ways of representing $\frac{1}{3}$ might you be able to find?

- Is there a limit to how many ways you might be able to find?

Pattern block fractions

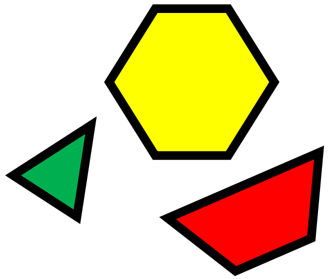


Create a design using 2 different pieces.

- If we call your whole design ‘1’, what is the value of each of the pieces?

- Can you convince the person beside you using reasoning.
- What do you notice about the relationship between the different pieces?

Pattern block fractions



Create a design where one hexagon is $\frac{3}{4}$ of the area.

How many different designs could you make?
Create them and then draw and label on isometric paper

Create a design where the hexagon is two-fifths of the area

Handfuls:

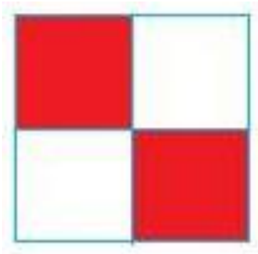
Take a large handful of pattern blocks and determine the value of the blocks:

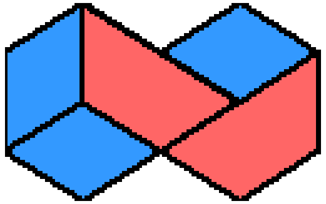
- If the hexagon is 1
- If the trapezium is 1
- If ….

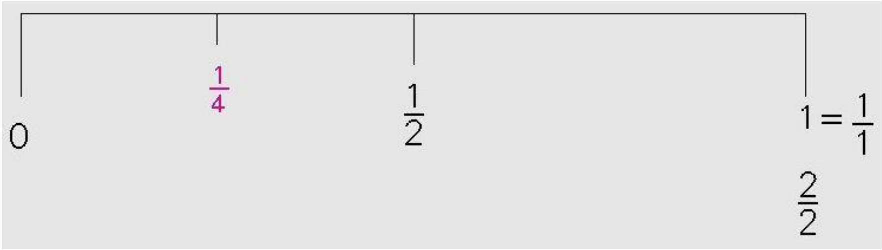
Using models and representations

Exploring use of models

Encourage students to model their thinking through the use of concrete materials, pictorial representations and language (spoken and written)







Models for fractions

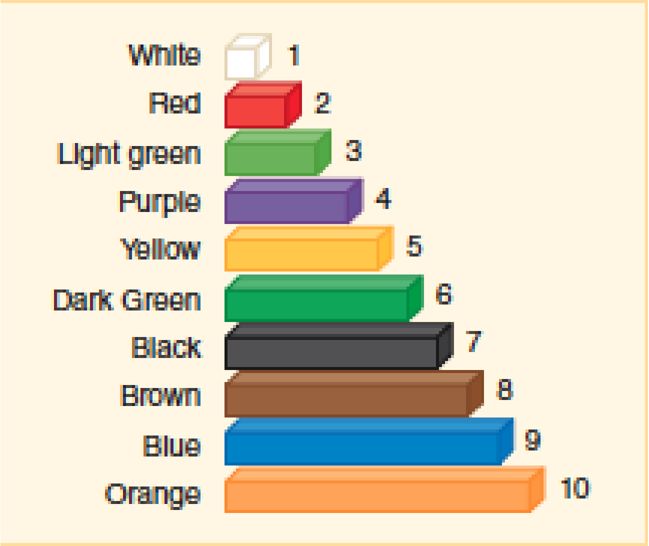
Model	What Defines the Whole	What Defines the Parts	What the Fraction Means
Area	The area of the defined region	Equal area	The part of the area covered as it relates to the whole unit
Length or number Line	The unit of distance or length	Equal distance/length	The location of a point in relation to 0 and other values on the number line
Set	Whatever value is determined as one set	Equal number of objects	The count of objects in the subset as it relates to the defined whole

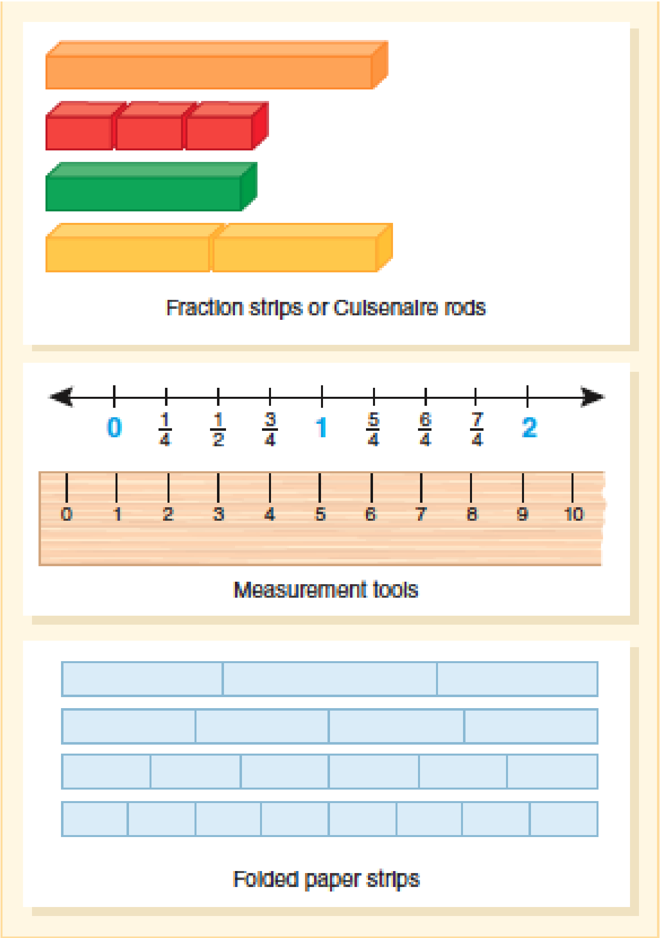
(Van de Walle et al., 2015)

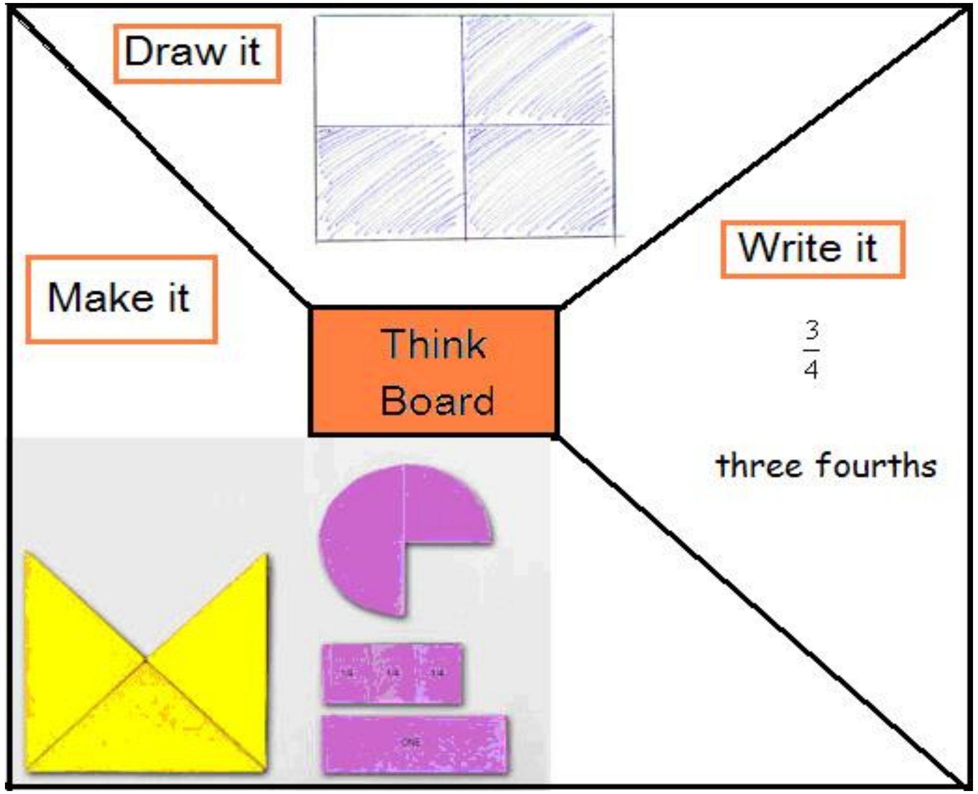
Fraction length models

Length models are physical materials that are compared on the basis of length

Number lines are subdivided.







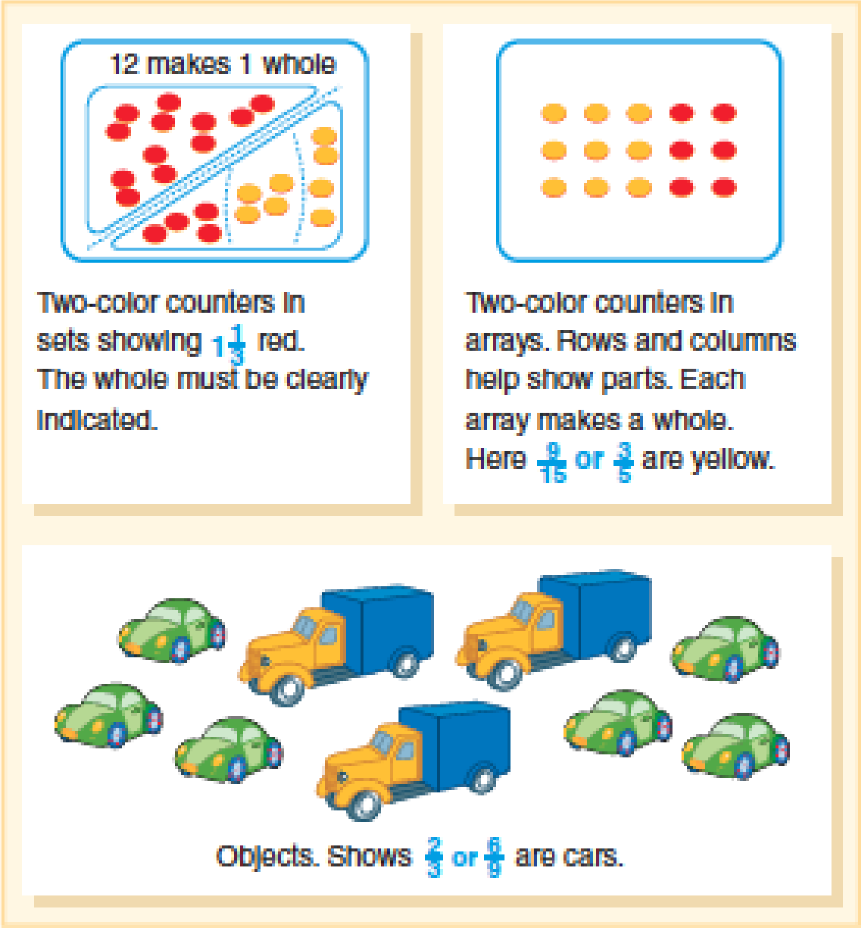
Helping students connect the information about number triad relationships

Using a modified 'think board' is an important strategy for developing quantity sense about fractions.

Be sure that the students use a range of models (set and area models) when representing the quantity associated with the fraction

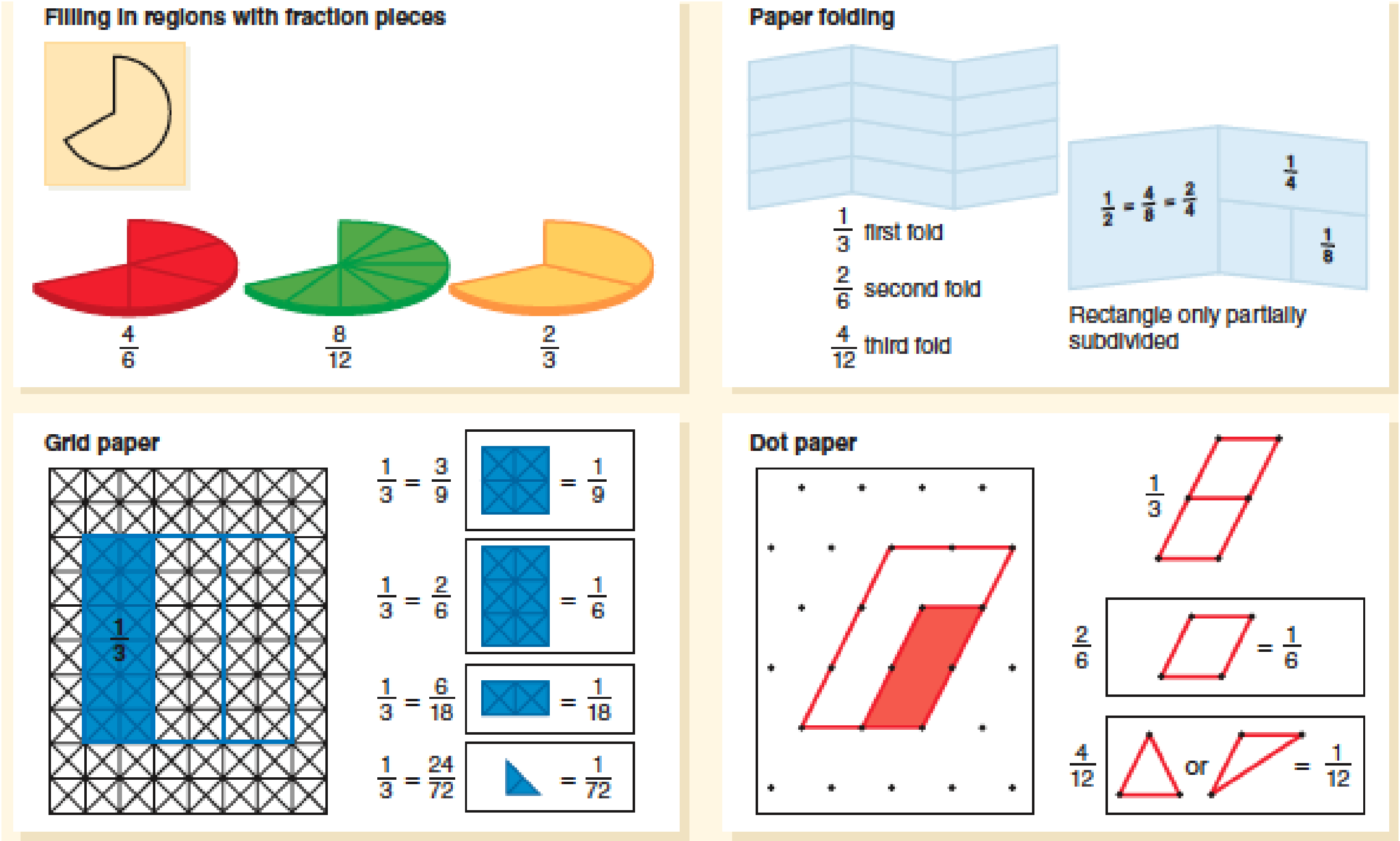
Fraction set models

Set models, the whole is understood to be a set of objects and the subsets of the whole make up fractional parts.



Conceptual focus on equivalence

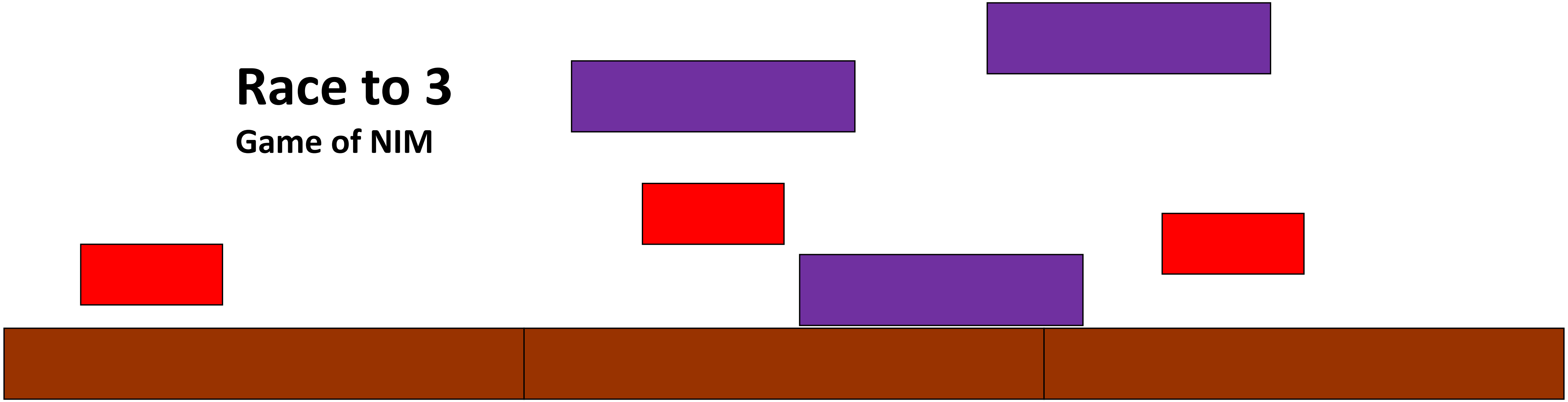
Area models for equivalent fractions help students create understanding



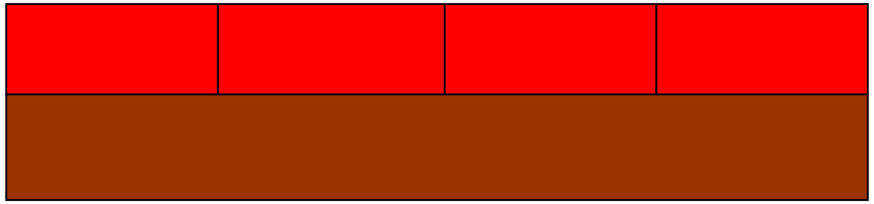
‘Cuisenaire fractions’

Race to 3

Game of NIM



What fraction of the brown rod is the red rod?



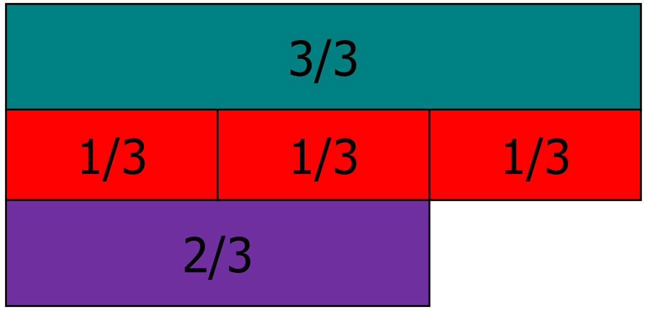
$$\frac{1}{4}$$

Cuisenaire questions

- What fraction of the brown rod is the red rod?
- If the pink (pink/purple) is two thirds, what is the whole?
- If the brown rod is $\frac{4}{3}$, what rod is one?
- If the dark green is $\frac{1}{2}$, what is $\frac{3}{4}$?
- If the blue rod is $1\frac{1}{2}$, what is $\frac{2}{3}$?

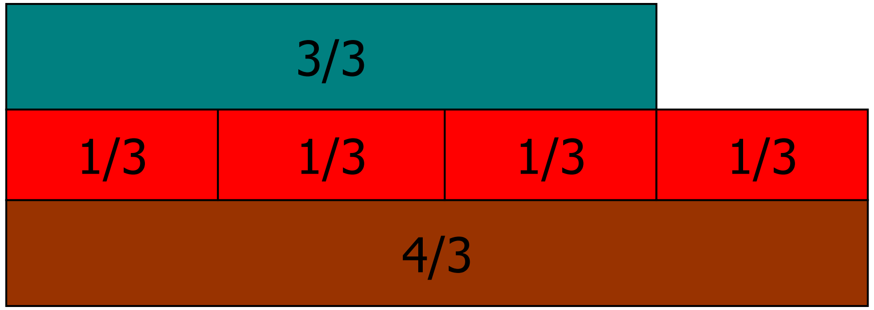
- What other questions might you have?

If pink/purple is two-thirds, what is one (1)?



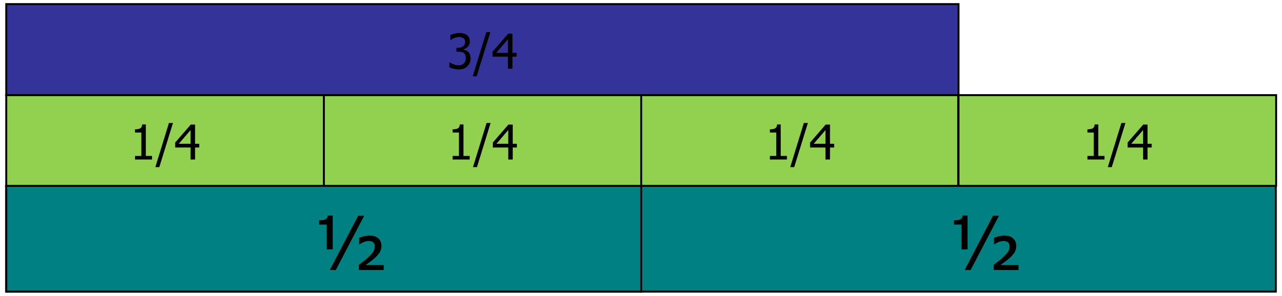
The dark green rod is 1

If brown is four-thirds, what rod is one (1)?



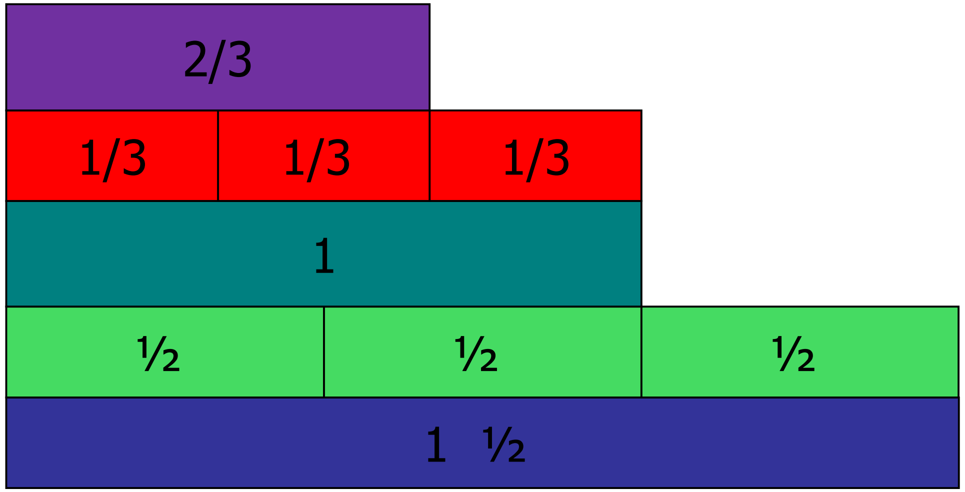
The dark green rod is one

If dark green is half, what is three-quarters?



The blue rod is $\frac{3}{4}$

If the blue rod is one and a half, what is two-thirds?



The purple/pink rod is $\frac{2}{3}$

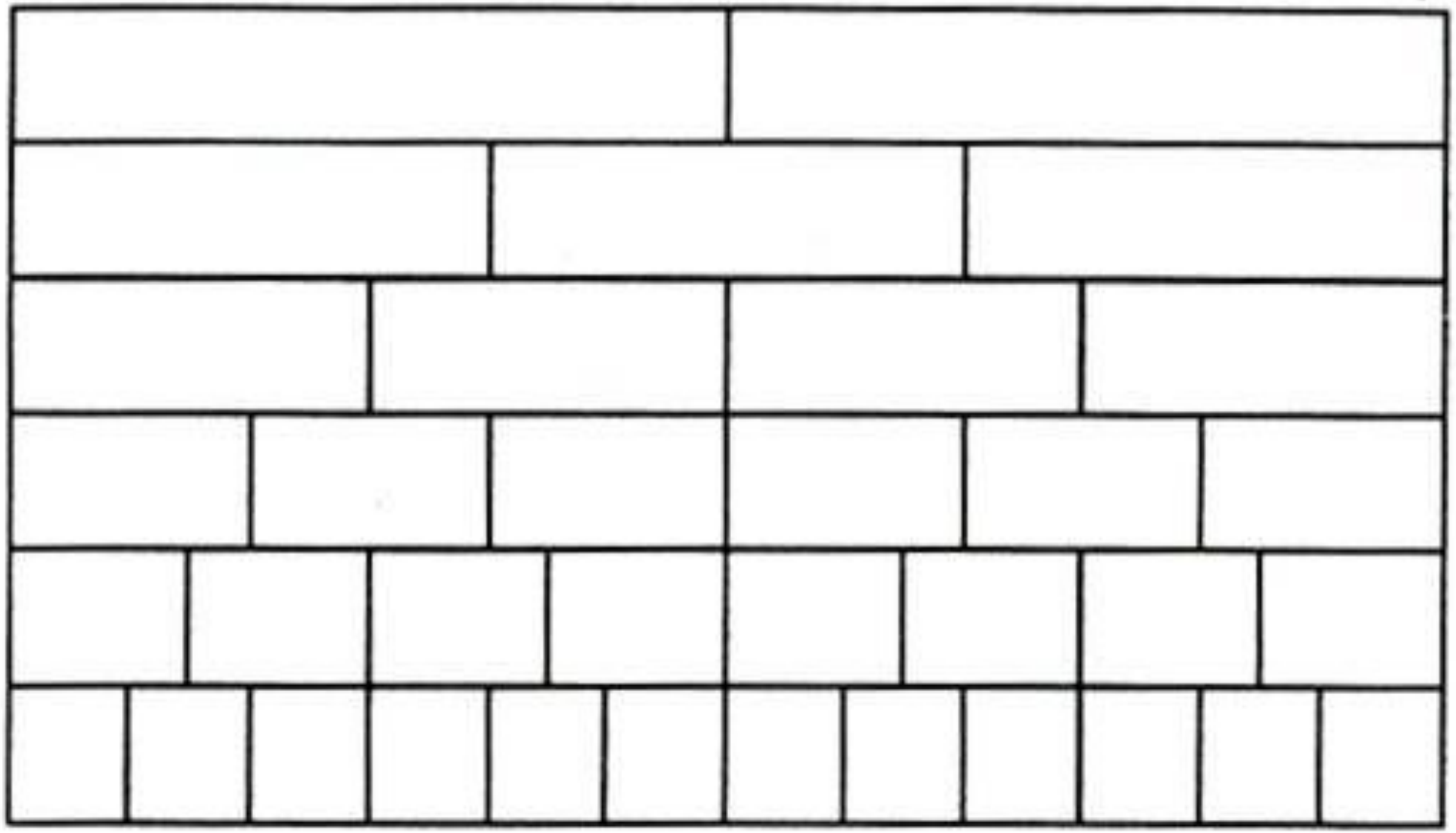
Cuisenaire fractions

Why might you use this learning assignment?

- What concepts does it support students to experience?

- What aspects might students find easier?
- What aspects might be more challenging?
- Why do you think this?

Colour in Fractions (Clarke & Roche, 2010)

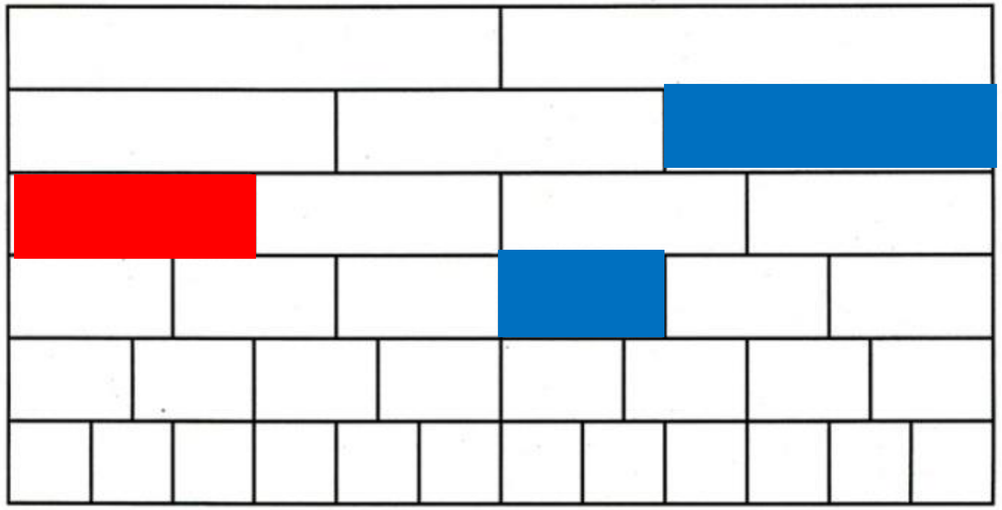


Colour in fractions

- Take turns in rolling both dice:
  - Dice 1: 1, 2, 2, 3, 3 ,4
  - Dice 2: $\frac{*}{2}, \frac{*}{3}, \frac{*}{4}, \frac{*}{6}, \frac{*}{8}, \frac{*}{12}$

- Each row represents one whole
- Use the numbers rolled to create a fraction.
  - For example 3 and $\frac{*}{6}$
  - You can colour in $\frac{3}{6}$ on one line.
    - **What other options might there be?**
- Each player needs to convince the other that what is shading is correct.
- If players are unable to use their turn they must ‘pass’
- The first player to colour their entire wall is the winner

Activity Sheet: Colour in Fractions



What I rolled	What I shaded
2/8	1/4
3/6	1/3 + 1/6

What I rolled	What I shaded

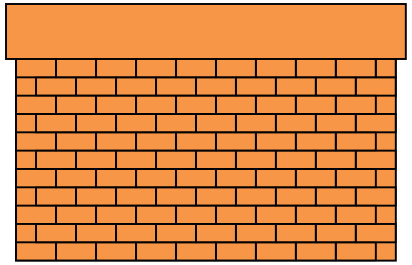
Moving between parts and wholes Susan Lamon

- Moving from the whole to the part

"Here is one chocolate block…show me a third of the block."

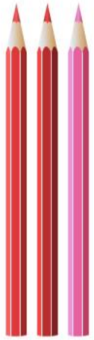
- Moving from the part to the whole

"Three-quarters of the brick wall has been built…show me the whole brick wall."



- Moving from the part to the part

"This is one-quarter of a pencil set…show me five-quarters of a set."



Robust definitions for the numerator and denominator

A change from...

What do the numbers mean?

$$\frac{3}{4}$$

The four is the number of parts you cut the whole into and the three is the number of parts you take from the whole

Can this explanation be generalised?

How about if we had an improper fraction like $\frac{7}{4}$?

What do the numbers mean?

$$\frac{3}{4}$$

The denominator (4) represents the **name or size of the part** (e.g., the four represents quarters and they have this name because 4 equal parts fill a whole) and numerator (3) is the **number of parts *of that name* or size**. *[three-quarters]*

$$\frac{7}{4}$$

Four is the name or size of the parts (quarters) and seven is the number of quarters. [seven-quarters]

Fraction pair interview

(Clarke & Roche, ACU)

Fractions Pair Interview (Clarke & Roche, ACU)

Interviewing as assessment

Using the fraction pair cards, you interview the student

The teacher will ask the student to choose which of the two fractions in the pair is the largest and ask for an explanation

The teacher asks each time:

Please point and tell me which is the larger fraction...How did you decide?

The teacher will record on the sheet the thinking strategy that was used by the student

Fraction Pairs Record Sheet

Name:
School:
Year Level:

Show the student each fraction pair card, one at a time.
Please point to the larger fraction…. How did you decide?
Don't allow use of paper and pencil.
Only continue with f-h if complete success in a-e

a. $\frac{3}{8}$ $\frac{7}{8}$

- **d the same and compares n**
- Benchmarking to ½ and/or 1
- Residual thinking (1/8 < 5/8)
- Other (satisfactory)

- Compares numerator only (7>3)
- Gap thinking (1 < 5)
- Smaller numbers mean bigger fractions
- Other (unsatisfactory)

b. $\frac{2}{4}$ $\frac{4}{8}$

- **Equivalent ("the same")**
- Other (satisfactory)

- "Higher" or "larger" numbers
- Gap thinking (2 < 4)
- Other (unsatisfactory)

c. $\frac{1}{2}$ $\frac{5}{8}$

- **Benchmarks to one half (5/8 > 1/2)**
- Converts to common denominator (5/8 > 4/8)
- Other (satisfactory)

- "Higher" or "larger" numbers
- Gap thinking (1 < 3)
- Other (unsatisfactory)

d. $\frac{2}{4}$ $\frac{4}{2}$

- **Equates to ½ and 2**
- Equates to a ½ and more than 1
- Converts to common denominator
- (8/4 > 2/4 or 4/2 > 1/2 etc)
- Other (satisfactory)

- "Both the same"
- Compares numerators or denominators (4 > 2)
- Improper fraction
- Other (unsatisfactory)

e. $\frac{4}{7}$ $\frac{4}{5}$

- **n the same and compares d**
- Converts to common denominator (28/35 >20/35)
- Benchmarks to ½ and 1
- Residual thinking (1/5 < 3/7)
- Other (satisfactory)

- More area (sometimes related to an image)
- Compares denominator only (7>5)
- Gap thinking (1< 3)
- Other (unsatisfactory)

f. $\frac{3}{7}$ $\frac{5}{8}$

- **Benchmarks to one half (3/7 < 1/2 & 5/8 > 1/2)**
- Converts to common denominator (35/56 > 24/56)
- Other (satisfactory)

- Residual thinking (3/8 < 4/7)
- "Higher" or "larger" numbers
- Gap thinking (3 < 4)
- Other (unsatisfactory)

g. $\frac{5}{6}$ $\frac{7}{8}$

- **Residual thinking (1/6 > 1/8)**
- Converts to common denominator (21/24 >20/24 or 42/48 > 40/48)
- Other (satisfactory)

- "Higher" or "larger" numbers
- Gap thinking (both have a gap of one)
- Other (unsatisfactory)

h. $\frac{3}{4}$ $\frac{7}{9}$

- **Residual thinking with equivalence (2/8 > 2/9)**
- Residual thinking (1/4 > 2/9) with some other proof
- Converts to common denominator (28/36 > 27/36)
- Other (satisfactory)

- "Higher" or "larger" numbers
- Gap thinking (1 < 2)
- Other (unsatisfactory)

Strategies that are noted above the line in each rectangle are considered "preferred strategies"

By preferred, we mean strategies which are built on conceptual understanding of fractions and their sizes

Benchmarking

Benchmarking is a thinking strategy that can be used to compare the quantity or size of two fractions (Clarke & Roche, 2009)

It involves the use of a third fraction or benchmark, usually 0, $\frac{1}{2}$ or 1.

When a student uses benchmarking, he/she will decide that $\frac{4}{5}$ is larger than $\frac{3}{7}$ because the latter fraction is closer to a half

Benchmarking

Where do these fractions belong?

Close to 0

$\frac{5}{9}$ $\frac{3}{100}$ $\frac{7}{8}$ $\frac{6}{14}$ $\frac{9}{4}$ $\frac{1}{15}$

$\frac{1}{14}$

$\frac{6}{7}$ $\frac{6}{4}$

$\frac{4}{5}$ $\frac{5}{2}$

Close to ½

Close to 1

Comparing fractions

Think about the two fractions

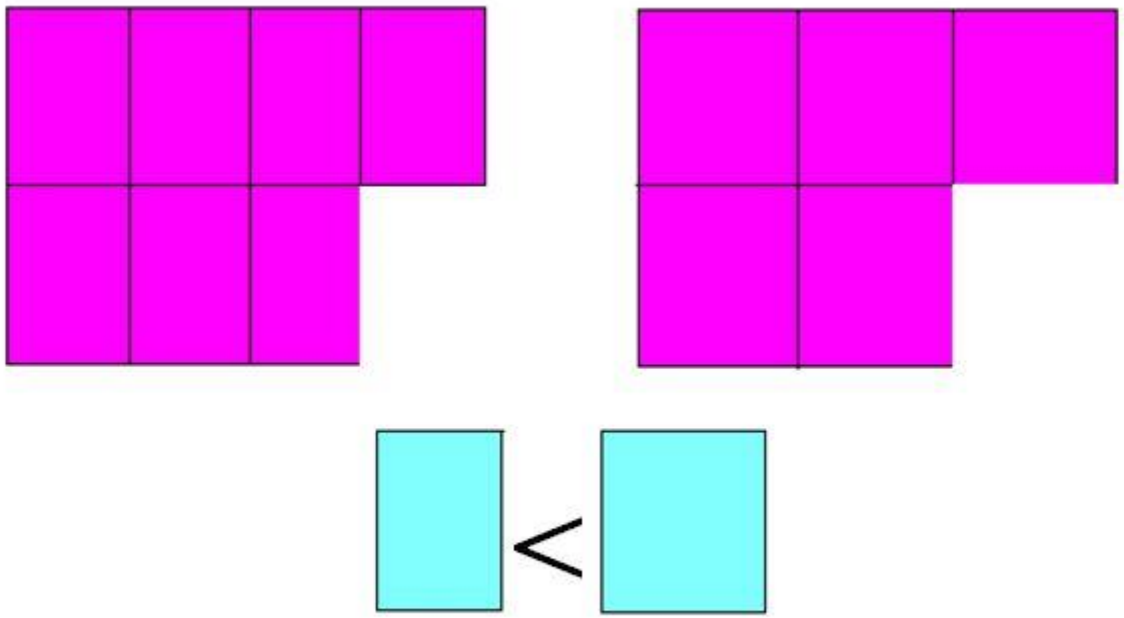
$$\frac{7}{8} \quad \frac{5}{6}$$

Which is larger and why?

Use two strategies to prove which is larger

Residual thinking

$\frac{7}{8}$ $\frac{5}{6}$



$$\frac{1}{8} < \frac{1}{6}$$

Residual thinking

$$\frac{7}{8} > \frac{5}{6}$$

1/8 is less than 1/6

therefore 7/8 is larger

Residual thinking relies on understanding the amount that is needed when building up to the whole

(Clarke & Roche, 2009)

Gap thinking
(common misconception)

$$\frac{5}{6} = \frac{7}{8}$$

"Only one piece or one number between 5 and 6 or 7 and 8 so this means that they are the same"

This is an example of using whole number thinking with rational numbers

(Pearn & Stephens, 2004)

Teaching considerations

1. Emphasise number sense and meaning of fractions
2. Emphasise that fractions are numbers and therefore quantities
3. Provide a variety of models and contexts, including examples, non-examples, and images that go beyond the prototype
4. Dedicate time for understanding of equivalence (concretely, symbolically)
5. Teach “fraction families” to help students build connections between fraction sizes, i.e., explore the “halving family” – half, quarter, eighths, sixteenths…explore the “thirding family” – third, ninths, twenty-sevenths…
6. Link fractions to key benchmarks and encourage estimation
7. Highlight comparison strategies that focus on conceptual understanding of fraction sizes

A change to....

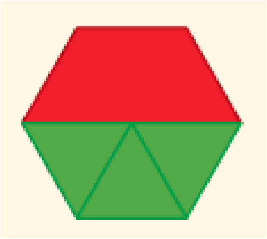
$$\frac{3}{4}$$

Denominator represents the ***name or size*** of the parts (e.g., quarters)

Numerator is the ***number of parts*** of that name or size (e.g., 3)

(Van de Walle et al., 2015)

Common misconceptions



- Thinking of numerator and denominator as separate and not as a single value
- Not recognizing equal-sized parts--thinking ¾ green instead of ½ green
- Thinking that fraction $\frac{1}{5}$ is smaller than $\frac{1}{10}$ because it has a smaller denominator
- Using the operation rules from whole numbers to compute with fractions
- Having only a limited number of images which are most likely the prototypical ones and generally area models (circle or square)

Fraction Numberlines

Peg and Tape Fractions

Fraction number lines

Materials: Number cards (0, 1, 2); fraction cards including improper fractions, rope, pegs

- Hand out the fraction cards (1 per pair)
- Ask students to talk about what they know about the fraction, focusing on its quantity and the representations that they think about and visualise
- Ask students to place the whole numbers on the clothesline, explaining reasons for their placement
- Invite pairs of students to place their fraction card on the line using the peg
- Encourage discussion about reasons for the placement of the card along the line